

# The Cosmological Constant in Calabi-Yau 3-fold Compactifications of the Horava-Witten Theory

S. P. de Alwis and N. Irges<sup>1</sup>

*Department of Physics, Box 390, University of Colorado, Boulder, CO 80309.*

## Abstract

Brane world scenarios offer a way of setting the cosmological constant to zero after supersymmetry breaking provided there is a sufficient number of adjustable integration constants/parameters. In the case of the Horava-Witten theory compactified on a Calabi-Yau threefold, we argue that it is difficult to find enough freedom to get a zero (or small) cosmological constant after supersymmetry breaking.

---

<sup>1</sup>e-mail: dealwis@pizero.colorado.edu, irges@pizero.colorado.edu

# 1 Introduction

One of the phenomenologically most relevant corners of the string/M-theory moduli space is the Horava-Witten theory [1]. It corresponds to compactifying M-theory on  $R^{10} \times S^1/Z_2$ , which results in an eleven dimensional “brane world”, with two ten dimensional branes sitting at the fixed points of the orbifold, each of them supporting an  $E_8$  gauge theory. Compactifying six of the dimensions of  $R^{10}$  on a Calabi-Yau three-fold (CY<sub>3</sub>), results in a five dimensional brane world with an  $\mathcal{N} = 2$  gauged supergravity in the bulk and an  $\mathcal{N} = 1$  supersymmetric  $E_8$  gauge theory on each three-brane at the fixed points. A detailed derivation of the five dimensional low energy effective action for these compactifications was given in [2].

Such five dimensional brane world compactifications, besides their obvious phenomenological relevance due to the four dimensional  $\mathcal{N} = 1$  gauge theories coupled to chiral matter, offer a means for setting the four dimensional cosmological constant (CC) [3] [4] to zero, provided there are adjustable integration constants/parameters. Indeed, in another class of brane world scenarios, namely compactifications of type II string theory on curved spaces (such as  $S^5$  or deformations of it) with the gauge theory coming from D-branes sitting at orientifold fixed planes, it is indeed possible to argue that one can get flat space after supersymmetry breaking by adjusting integration constants [5]. In this paper, we discuss the same issue in the context of the Horava-Witten theory.

It was shown in the third paper of reference [1] that a consistent compactification of M-theory ala Horava-Witten on a Calabi-Yau three-fold requires a non-trivial non-zero mode flux for the four form field strength  $\mathcal{G}$ . This corresponds to wrapped M5-branes around non-trivial  $(1, 1)$  cycles of the Calabi-Yau and one of the consequences of this background flux is a non-zero potential for the volume and shape moduli [2].

One could also consider the existence, for example of instanton corrections coming from (Euclidean) M5-branes wrapped around the Calabi-Yau manifold. Indeed, this seems to be the case, as discussed in [6], even though the exact form of this correction to the potential is not known. What we do know, however, is that the volume modulus dependence is the same as for the potential from the compactification, a fact that can be easily justified once we realize that both contributions can be seen as arising from gauging Abelian isometries of the universal hypermultiplet coset space [7], [6]. The total potential due to M5-branes can be therefore written as

$$V_{M5} = e^{2\varphi}(V_{(1,1)} + V_{(3,3)}), \quad (1)$$

where  $\varphi$  is the volume modulus,  $V_{(1,1)}$  and  $V_{(3,3)}$  correspond to M5-branes wrapped around  $(1, 1)$  and the  $(3, 3)$  cycle respectively.

In section 2, we rederive  $V_{(1,1)}$  following [2] ignoring for simplicity instanton contributions. In section 3 we discuss the cosmological constant problem from the point of view of the five dimensional brane world after supersymmetry breaking. We will conclude that after supersymmetry breaking there do not exist flat (or nearly flat) brane solutions to these equations. In section 4, we summarize our conclusions.

## 2 Horava-Witten on $CY_3$

The bosonic part of the low energy effective action of  $M$ -theory is the bosonic part of the eleven dimensional supergravity of [8]:

$$S = -\frac{1}{2\kappa^{(11)^2}} \left\{ \int d^{11}x \sqrt{-g^{(11)}} \mathcal{R}^{(11)} - \frac{1}{2} \int (\mathcal{G} \wedge *\mathcal{G} + \frac{1}{3} \mathcal{C} \wedge \mathcal{G} \wedge \mathcal{G}) \right\}, \quad (2)$$

with  $\mathcal{C}$  the 3-form of 11 dimensional supergravity and  $\mathcal{G} = d\mathcal{C}$ . This action, when supplemented by appropriate boundary terms, describes the (low energy) strong coupling limit of the heterotic string theory, of Horava and Witten.

We first consider the effective action obtained by reducing (2) on a Calabi-Yau 3-fold [9]. The appropriate metric ansatz is

$$ds^{(11)^2} = ds_{str}^{(5)^2} + ds_{CY}^{(6)^2}. \quad (3)$$

The subscript “*str*” indicates string frame and superscripts indicate the dimensionality. We assume, following [1] [2], that the “standard embedding” requires in the Horava-Witten picture a non-trivial  $\mathcal{G}$ -flux of the form

$$\mathcal{G} = \frac{i}{2\mathcal{V}} n_i G^{ij} (*\omega_j), \quad (4)$$

where the  $n_i$  are integers. The form of the  $\mathcal{G}$ -flux for non-standard embeddings and additional five branes in the bulk is similar; the only modification is that the  $n_i$  in the above is replaced by a sum over fluxes [10]. The low energy spectrum includes the five dimensional graviton multiplet,  $h^{1,1} - 1$  vector multiplets containing one shape modulus scalar ( $b^i$ ) each ( $i = 1 \dots h^{1,1}$  but there is one constraint, see below), and the universal hypermultiplet that includes the volume modulus ( $\varphi$ ), the scalar dual to the five dimensional three-form ( $\sigma$ ) and a pair of complex scalars ( $\xi$  and  $\bar{\xi}$ ). The metric on the vector moduli space is

$$G_{ij}(a) = \frac{i}{2\mathcal{V}} \int_{CY} \omega_i \wedge *\omega_j = -\frac{1}{2} \partial_i \partial_j \ln \mathcal{V}(a), \quad (5)$$

The Kähler form is  $J = a^i \omega_i$ , with the  $\omega_i$  a basis of  $h^{1,1}$  2-forms and  $a^i$  are the  $h^{1,1}$  Kähler moduli of the Calabi-Yau. The Calabi-Yau volume  $\mathcal{V}$  is

$$\mathcal{V}(a) = \frac{1}{3!} \int_{CY} J \wedge J \wedge J = \frac{1}{6} c_{ijk} a^i a^j a^k, \quad (6)$$

with  $c_{ijk}$  the intersection numbers of the Calabi-Yau.

For our purposes, it is sufficient to consider the part of the effective action which is 5D gravity coupled to the scalars of the vector multiplets and the volume modulus (breathing mode) of the universal hypermultiplet. The corresponding five dimensional string frame bulk effective action resulting from the first two terms of (2), is

$$S = -\frac{1}{2\kappa^{(5)^2}} \int d^5x \sqrt{-g_{str}^{(5)}} \left[ \mathcal{V} \mathcal{R}^{(5)} + (\mathcal{V} G_{ij}(a) + \partial_i \partial_j \mathcal{V}) \partial_I a^i \partial^I a^j - \frac{1}{4\mathcal{V}} G^{ij}(a) n_i n_j \right]. \quad (7)$$

The five dimensional index is  $I = \{\mu, r\}$ . We have neglected the terms coming from the Chern-Simons term in (2) since they are irrelevant to our discussion. After the Weyl rescaling  $ds_E^{(5)^2} =$

$\mathcal{V}^{\frac{2}{3}} ds_{str}^{(5)2}$  and separation of the volume modulus from the shape moduli which can be done by defining  $b^i = a^i \mathcal{V}^{-1/3}$ , we arrive at a bulk action of the form

$$S_{bulk} = -\frac{1}{2\kappa^{(5)2}} \int d^5x \sqrt{-g_E^{(5)}} \left[ \mathcal{R}^{(5)} - G_{ij}(b) \partial_I b^i \partial^I b^j - \frac{1}{2} \partial_I \varphi \partial^I \varphi - \frac{1}{4} e^{2\varphi} G^{ij}(b) n_i n_j + \lambda (c_{ijk} b^i b^j b^k - 6) \right], \quad (8)$$

where we have defined  $\mathcal{V} = e^{-\varphi}$  and  $\lambda$  is a Lagrange multiplier. This is not the whole relevant Horava-Witten 5D action, because we have not taken into account yet the Horava-Witten Wall/M5-brane action sitting at the fixed points of the orbifold. The additional brane terms are

$$S_{branes} = \frac{1}{2\kappa^{(5)2}} \int d^4x \sqrt{-g_E^{(4)}} T_1(\varphi) \delta(r) + \frac{1}{2\kappa^{(5)2}} \int d^4x \sqrt{-g_E^{(4)}} T_2(\varphi) \delta(r - \pi R), \quad (9)$$

with  $g_E^{(4)}$  the induced metric on the brane (we will assume static gauge and ignore fluctuations) and  $T_{1,2}$  are the tensions of the branes. As we mentioned, these 3-branes arise from the M5-branes of the original 11 dimensional theory that are wrapped around non-trivial 2-cycles of the Calabi-Yau and the brane tensions at the string scale, i.e. before supersymmetry breaking, are simply proportional to the volume of the 2-cycle [2], [11]:

$$T_1 = -T_2(\varphi) = \frac{2}{1!} \int J = 2e^\varphi \zeta. \quad (10)$$

Defining  $n^2 \equiv G^{ij} n_i n_j = n_i n^i$  and eliminating the Lagrange multiplier, we can write the equations of motion for the  $b^i$  as

$$\begin{aligned} & -\frac{1}{4} e^{2\varphi} (\partial_k n^2) + \frac{1}{6} e^{2\varphi} b_k b^l (\partial_l n^2) \\ & -\frac{3}{2} c_{ijk} (\nabla_\alpha b^i) (\nabla^\alpha b^j) + \frac{2}{3} c_{ijl} b^l b_k (\nabla_\alpha b^i) (\nabla^\alpha b^j) + \frac{4}{3} b_k b_i (\nabla_\alpha \nabla^\alpha b^i) \\ & = e^\varphi (\delta(r) - \delta(r - \pi R)) \left( \frac{4}{3} n_l b^l b_k - 2n_k \right). \end{aligned} \quad (11)$$

To simplify these equations, we will assume that the  $b^i$  take constant values in the vacuum. All the terms with the covariant derivatives drop out, the equations of motion for  $b^i$  decouple from the equation of motion for  $\varphi$  and then the unique ansatz that solves the remaining of (11) is [2], [11]

$$\zeta b_k = \frac{3}{2} n_k, \quad (12)$$

where  $\zeta \equiv n_l b^l$ . By solving the above system of equations, we obtain the values that the squashing modes take in the vacuum. Notice that for the ansatz (12), the term multiplying the  $\delta$ -functions in (11) vanishes, which is a necessary condition for a consistent solution with constant  $b^i$ . Now we turn to the breathing mode  $\varphi$ .

The bulk action, in the Einstein frame, with all the moduli besides the breathing mode  $\varphi$  stabilized, is

$$S_{bulk} = -\frac{1}{2\kappa^{(5)2}} \int d^5x \sqrt{-g_E^{(5)}} \left[ \mathcal{R}^{(5)} - \frac{1}{2} (\partial\varphi)^2 - \frac{1}{6} \zeta^2 e^{2\varphi} \right]. \quad (13)$$

In the above, the constant  $\zeta^2$  in front of the potential is determined in terms of the integer  $n_i$  (therefore it is not a continuous quantity) and it corresponds to contributions from fluxes associated with wrapping M5 branes around the non trivial (1, 1) cycles of the Calabi Yau. From the 5D

gauged supergravity point of view, the potential comes from gauging the  $U(1)$  isometry of the universal hypermultiplet moduli space  $SU(2,1)/U(2)$  corresponding to a shift symmetry of the scalar  $\sigma$ . It is interesting to note here that the universal hypermultiplet moduli space has more  $U(1)$  isometries that could be gauged [7], associated with shift symmetries of  $\xi$  and  $\bar{\xi}$ . Gauging all of the  $U(1)$  isometries, in principle, can produce additional terms in the potential. From the M-theory point of view, the additional pieces correspond to M5-branes wrapped around the  $(3,3)$  cycle (i.e. the whole Calabi-Yau) and/or to M2-branes wrapped around  $(3,0)$  and  $(0,3)$  cycles of the Calabi-Yau [6]. The potential in (13) is therefore the potential for  $CY_3$  compactifications of the Horava-Witten theory for constant shape moduli and without an M5/M2 instanton gas. In the following, we will neglect instanton contributions, since our subsequent arguments about the cosmological constant are not affected by their presence.

### 3 Supersymmetry Breaking and the Cosmological Constant

The equations of motion from action (13), for  $\varphi = \varphi(r)$  and for

$$ds_E^{(5)2} = e^{2A(r)} \eta_{\mu\nu} dx^\mu dx^\nu + (dr)^2, \quad (14)$$

are

$$\varphi'' + 4A'\varphi' = \frac{\partial V(\varphi)}{\partial \varphi} \quad (15)$$

$$A'' = -\frac{1}{6}\varphi'^2 \quad (16)$$

$$A'^2 = -\frac{1}{12}V(\varphi) + \frac{1}{24}\varphi'^2, \quad (17)$$

where  $V(\varphi) \equiv e^{2\varphi}V_{(1,1)} = e^{2\varphi}\frac{1}{6}\zeta^2$ . The prime denotes differentiation with respect to  $r$ . The first order set of equations corresponding to the above second order set, in terms of the superpotential  $W$ , is

$$\varphi' = \frac{\partial W}{\partial \varphi} \quad (18)$$

$$A' = -\frac{1}{6}W \quad (19)$$

$$V(\varphi) = \frac{1}{2}\left(\frac{\partial W}{\partial \varphi}\right)^2 - \frac{1}{3}W^2. \quad (20)$$

We note here that the proper ansatz for the five dimensional metric would be to replace the flat Minkowski metric  $\eta_{\mu\nu}$  by  $g_{\mu\nu}^{(A)dS}$  (i.e. a metric for deSitter or Anti-deSitter four-space) but since the (possibly) measured value of the CC is many tens of orders of magnitude smaller than all relevant scales in this analysis, this is not a meaningful difference. We can rewrite (20) as

$$\left(\frac{\dot{W}}{\sqrt{2V}}\right)^2 - \left(\frac{W}{\sqrt{3V}}\right)^2 = 1, \quad (21)$$

where the dot stands for differentiation with respect to  $\varphi$ . The ansatz

$$\dot{W} = \sqrt{2V} \cosh f(\varphi) \quad \text{and} \quad W = \sqrt{3V} \sinh f(\varphi) \quad (22)$$

allows us to separate variables, so that combining the above, we can integrate over  $\varphi$  and  $f$ :

$$\int d\varphi = \int \frac{df}{\sqrt{\frac{2}{3}} - \tanh f}. \quad (23)$$

The integral yields

$$c_1 e^{-\varphi} = (\tanh f - \sqrt{\frac{2}{3}})^3 \frac{(\tanh f + 1) \sqrt{\frac{3}{2}} (1 - \sqrt{\frac{2}{3}})}{(\tanh f - 1) \sqrt{\frac{3}{2}} (1 + \sqrt{\frac{2}{3}})}, \quad (24)$$

with  $c_1$  an integration constant. Now equations (18) and (19) can be integrated as follows:

$$r + c_2 = \frac{\sqrt{3}}{\zeta} \int d\varphi e^{-\varphi} \sqrt{1 - \tanh^2 f(\varphi)} \quad (25)$$

and

$$A = -\frac{1}{6} \sqrt{\frac{3}{2}} \int d\varphi \tanh f(\varphi). \quad (26)$$

The method therefore to find a solution is to invert (24) for  $\tanh f$  in terms of  $\varphi$ , substitute into (25) and (26) and integrate. Then, to find the expression for  $\varphi(r)$ , invert (25) and finally use this to obtain the expression for  $A(r)$ .

The construction of the solution suited for an orbifold is described in detail for example in [5]. Consistency of our flat domain wall ansatz with the vanishing of the total 4 dimensional CC, amounts to satisfying the following jump conditions:

$$2\varphi'(r) = +\frac{\partial T_1}{\partial \varphi}(\varphi(r)) \big|_{r=0} \quad (27)$$

$$2A'(r) = -\frac{1}{6} T_1(\varphi(r)) \big|_{r=0} \quad (28)$$

$$2\varphi'(r) = -\frac{\partial T_2}{\partial \varphi}(\varphi(r)) \big|_{r=\pi R} \quad (29)$$

$$2A'(r) = +\frac{1}{6} T_2(\varphi(r)) \big|_{r=\pi R}. \quad (30)$$

Equations (27), (29), (28) and (30) is the system of equations that has to be satisfied in a model with vanishing CC. To satisfy these, we have the two integration constants  $c_1$  and  $c_2$ , the size of the orbifold  $R$  and the discrete values of the  $n_i$  that determine  $\zeta^2$ . At the supersymmetric point, we can rewrite (27)-(30) in a more convenient form using (22):

$$\cosh f(\varphi(0)) = \sqrt{3} \quad (31)$$

$$\sinh f(\varphi(0)) = \sqrt{2} \quad (32)$$

$$\cosh f(\varphi(\pi R)) = -\sqrt{3} \quad (33)$$

$$\sinh f(\varphi(\pi R)) = -\sqrt{2}. \quad (34)$$

A simple solution to the above can be found if we take  $W = \zeta e^\varphi$ , which is a solution to (20). Then,

$$\cosh f(\varphi(r)) = \pm\sqrt{3} \quad \text{and} \quad \sinh f(\varphi(r)) = \pm\sqrt{2}, \quad (35)$$

and the equations of motion (25) and (26) can be solved easily, yielding

$$\varphi(r) = -\ln(\zeta|r| + c) \quad \text{and} \quad A(r) = \frac{1}{6}\ln(\zeta|r| + c), \quad (36)$$

with  $c \equiv \zeta c_2$  the (only) integration constant. This is a trivial solution, trivial in the sense that the jump conditions are satisfied identically, without restriction on  $\zeta$  and the integration constant  $c$ . In fact, since  $\tanh f(\varphi(r=0, \pi R)) = \sqrt{\frac{2}{3}}$  at the supersymmetric point, from (23) we see that at the supersymmetric point, this is actually the only possible solution.<sup>1</sup> Turning this around, we conclude that after supersymmetry breaking, the simple ansatz  $W = \zeta e^\varphi$  can not be used anymore to satisfy (27)-(30). But this is expected, since in general it is not possible to satisfy the four jump equations with the three parameters  $\zeta$ ,  $c$  and  $R$  (even if  $\zeta$  were continuous). Thus, for a consistent model with zero cosmological constant away from the supersymmetric point, we have to look for more general solutions to (20).

After supersymmetry breaking (on the brane), the brane tensions get renormalized as

$$T_1(\varphi) \rightarrow 2\zeta e^\varphi(1 + \epsilon\psi_1(\zeta e^\varphi)), \quad T_2(\varphi) \rightarrow 2\zeta e^\varphi(1 + \epsilon\psi_2(\zeta e^\varphi)) \quad (37)$$

with  $\epsilon$  being a small parameter characterizing the size of supersymmetry breaking. The scalar potential on the other hand, at least at the 5D level, remains the same [13].

The jump conditions therefore, after supersymmetry breaking become

$$\cosh f(\varphi(0)) = \sqrt{3}\left(1 + \epsilon\frac{d}{dx}(x\psi_1(x))|_{x=x(0)}\right) \quad (38)$$

$$\sinh f(\varphi(0)) = \sqrt{2}\left(1 + \epsilon\psi_1(x)|_{x=x(0)}\right) \quad (39)$$

$$\cosh f(\varphi(\pi R)) = -\sqrt{3}\left(1 + \epsilon\frac{d}{dx}(x\psi_2(x))|_{x=x(\pi R)}\right) \quad (40)$$

$$\sinh f(\varphi(\pi R)) = -\sqrt{2}\left(1 + \epsilon\psi_2(x)|_{x=x(\pi R)}\right), \quad (41)$$

where  $x = \zeta e^\varphi$ . Now let us recall that the difference of  $\cosh f$  ( $\sinh f$ ) from  $\sqrt{3}$  ( $\sqrt{2}$ ) vanishes like  $c_1$  which is of  $O(\epsilon)$ . Thus we may write the above equations as relations between  $O(1)$  functions. Now in accordance with the general argument of [4], the integration constants  $c_1, c_2$  may be adjusted to satisfy say the first two equations above. In addition the distance  $\pi R$  may be freely adjusted to satisfy one more matching condition leaving us with one more condition to satisfy. To satisfy the fourth condition then requires a fine tuning of the parameters in the bulk potential. In our case this means a fine tuning of the quantity  $\zeta$ . However, as noted earlier, the latter is completely determined in terms of integers  $n_i$  governing the fluxes and the integers  $d_{ijk}$  and  $h_{11}$  of the Calabi-Yau manifold. There is no way these can be chosen to satisfy this last equation since any change of these integers results in a  $O(1)$  change of the functions  $\psi_{1,2}$  and it would be a miracle if for any choice of the integers the condition could be satisfied<sup>2</sup>. In other words there is no adjustable continuous parameter that could be tuned to satisfy the equation.

---

<sup>1</sup>In general, even for compactifications on curved spaces such as type IIB on (squashed)  $S^5$  for example, such a trivial solution is always possible if the brane tension is taken to be  $W$  [12].

<sup>2</sup>Of course one does not require exact satisfaction of the matching conditions. Strictly speaking all we need is that the cosmological constant on the brane be of  $O(10^{-120}M_P^4)$ . This would still imply an adjustment of parameters to this accuracy as explained in [4] and is clearly ruled out in the present situation.

Let us now discuss possible generalizations. One possible non-vanishing supersymmetric correction to the scalar potential comes from the instanton gas obtained from wrapping M5-branes around the Calabi-Yau manifold or M2-branes wrapping around 3-cycles<sup>3</sup>. The potential would then be as in (1) plus an analogous contribution coming from M2-branes. Clearly, the original problem associated with the discreteness of the fourth adjustable parameter still remains since we are still talking about contributions that are completely determined by a set of integers.

Another possible generalization would be to look for solutions to the 5D equations of motion (11) with  $r$ -dependent  $b^i$ . Such a solution, for the supersymmetric case, was found in [2]. After supersymmetry breaking, we would have to satisfy two additional jump conditions for each  $b^i$ , which is possible, since we get two additional integration constants for each  $b^i$  from their equations of motion. However, as before, we still have one fine tuning to do and the previous argument, that since there is no continuous parameter there is no possibility of doing this, still applies.

## 4 Conclusions

We have discussed in this paper the two brane scenario coming from the Horava-Witten theory compactified on a Calabi-Yau manifold obtained by Lukas et al [2]. Those authors discussed the supersymmetry preserving case when there is sufficient degeneracy in the system of equations to satisfy the matching conditions without any adjustment of integration constants. When supersymmetry is broken however, as has been discussed in earlier work, we need four adjustable parameters (integration constants). We have shown explicitly that how these four parameters arise in the case at hand but that since one of the parameters necessarily takes discrete values, it does not seem possible to tune the cosmological constant to zero (or a small value).

The situation discussed in this paper is to be contrasted with the one obtained for compactifications on Ricci non-flat manifolds, as for example in IIB string theory on a (squashed)  $S^5$ , where one has two exponentials in the potential for the breathing mode. In these compactifications, besides a discrete parameter like  $\zeta$  which is also present, (coming from turning on five-form flux) there is an additional continuous parameter not present in Calabi-Yau compactifications. It is associated with the Ricci curvature of the compact space. Thus, in such cases, it is in principle always possible to satisfy the jump conditions and readjust integration constants by an arbitrarily small quantity after supersymmetry breaking to get flat brane solutions i.e. a cosmological constant that is zero<sup>4</sup> after supersymmetry breaking. Of course in the absence of a theory of integration constants one cannot claim to have solved the cosmological constant problem even in the case of compactifications on Ricci-non-flat manifolds. All one could claim there is that one has sufficient freedom to get a flat space solution. In the Ricci flat case what we have argued is that this freedom does not seem to exist.

## Acknowledgements

This work is partially supported by the Department of Energy contract No. DE-FG02-91-ER-

---

<sup>3</sup>It is not clear whether there are actually non-zero contributions due to transverse M2 branes parallel to the walls since there is no S-dual analog in the weak coupling i.e. the heterotic string limit.

<sup>4</sup>Or indeed of order  $O((10^{-3}eV)^4)$  as seems to be fashionable these days!



## References

- [1] P. Horava and E. Witten, “Heterotic and type I string dynamics from eleven dimensions”, Nucl. Phys. **B460** (1996) 506, [hep-th/9510209];  
 “Eleven-Dimensional Supergravity on a Manifold with Boundary”, Nucl. Phys. **B475** (1996) 94, [hep-th/9603142];  
 E. Witten, “Strong Coupling Expansion Of Calabi-Yau Compactification”, Nucl. Phys. **B471** (1996) 135, [hep-th/9602070].
- [2] A. Lukas, B. A. Ovrut, K. S. Stelle and D. Waldram, ”Heterotic M-theory in five-dimensions”, Nucl. Phys. **B552** (1999) 246, [hep-th/9806051].
- [3] L. Randall and R. Sundrum, “A large mass hierarchy from a small extra dimension”, Phys. Rev. Lett. **83** (1999) 3370, [hep-ph/9905221].
- [4] O. DeWolfe, D. Z. Freedman, S. S. Gubser and A. Karch, “Modeling the fifth dimension with scalars and gravity”, hep-th/9909134.
- [5] S. P. de Alwis, “Brane world scenarios and the cosmological constant”, hep-th/0002174;  
 S. P. de Alwis, A. T. Flournoy and N. Irges, ”Brane worlds, the cosmological constant and string theory”, hep-th/0004125;
- [6] M. Gutperle and M. Spalinski, “Supergravity instantons and the universal hypermultiplet”, hep-th/0005068;  
 K. Berhndt, “A non-singular infrared flow from  $D = 5$  gauged supergravity”, hep-th/0005185.
- [7] S. Ferrara and S. Sabharwal, “Quaternionic manifolds for type II superstring vacua of Calabi-Yau spaces”, Nucl. Phys. **B332** (1990) 317;  
 A. Strominger, “Loop corrections to the universal hypermultiplet”, Phys. Lett. **B421** (1998) 139, [hep-th/9706195].
- [8] E. Cremmer, B. Julia and J. Scherk, ”Supergravity theory in eleven dimensions”, Phys. Lett. **B76** (1978) 409.
- [9] A. C. Cadavid, A. Ceresole, R. D’Auria and S. Ferrara, ”11-dimensional supergravity compactified on Calabi-Yau threefolds”, Phys. Lett. **B357** (1995) 76.
- [10] A. Lukas, B. A. Ovrut, K. S. Stelle and D. Waldram, “Non-standard embedding and five-branes in heterotic M-theory”, Phys. Rev. **D59** (1999) 106005, [hep-th/9808101].
- [11] M. Spalinski and T. R. Taylor, “Branes and fluxes in D=5 Calabi-Yau compactifications of M-theory”, hep-th/0004095.

- [12] E. Bergshoeff, R. Kallosh and A. Van Proyen, “Supersymmetry in singular spaces”, hep-th/0007044.
- [13] A. Lukas, B. A. Ovrut and D. Waldram, ”Five-branes and supersymmetry breaking in M-theory”, **JHEP** 9904 (1999) 009, [hep-th/9901017].